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A note on vortex streets behind circular cylinders at low Reynolds numbers

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A discussion is given of the current state of knowledge of vortex streets behind circular cylinders in the Reynolds number range 50 to 160. This was prompted by Gaster's (1969) report that he could not find the transition at a Reynolds number of about 90 observed by Tritton (1959) and Berger (1964*a*). A further brief experiment confirming the existence of the transition is described Reasons for rejecting Gaster's interpretation are advanced. Possible (mutually alternative) explanations of the discrepant observations are suggested.

1. Introduction

In 1959 Tritton reported that the vortex street behind a circular cylinder undergoes a transition at a Reynolds number of about 90. The principal features of this transition were, first, a small discontinuity in the Reynolds number–Stouhal number (velocity–frequency) dependence, and secondly, the occurrence of irregular vortex streets when the Reynolds number was set close to the transitional value.

Gaster (1969) reports that he was unable to repeat the observation of the transition. (He was using the vortex street behind a straight cylinder as a speed calibration during experiments on vortex streets behind tapered cylinders.) He also suggests tentatively that the irregular vortex streets were the consequence of some non-uniformity in the flow.

In my opinion, Gaster's comments give a misleading impression of the present state of knowledge about vortex streets at low Reynolds numbers. They ignore some important pieces of information, particularly the results obtained by Berger (1964*a*) (although Gaster does refer to Berger's paper in another context). The purposes of the present note are to collect together the relevant information, to give reasons why Gaster's interpretation of my observations is unacceptable, and to present briefly the results of a short experiment to test the repeatability of these observations.

2. Survey of existing information

The flow under discussion is that past a circular cylinder of diameter D with its axis normal to the free stream velocity U in the Reynolds number range of approximately 50 to 160 ($R = UD/\nu$). This extends from the first appearance of a vortex street to the stage when the vortex street starts to become unstable as it travels downstream. This range was designated by Roshko (1954), who suggested an expression for the dependence of the vortex street frequency n, non-dimensionalized as the Strouhal number ($S = nD/U$) or as the parameter $F = SR$, on the Reynolds number.

The experiments reported in Tritton (1959) indicated that the expression should actually be divided into two with a small discontinuity between them. The corresponding flows were called the 'low-speed mode' (applying for Reynolds numbers below about 105) and the 'high-speed mode' (for Reynolds numbers above about 80, so that there was some overlap). These names will be retained here, although subsequent work has shown that they are an over-simplification. The role of these observations in our understanding of vortex street behaviour is discussed by Marris (1964). Tritton offered the interpretation that the discontinuity corresponded to a change in the origin of the vortex street from wake instability to shedding of attached eddies. This fits in with various bits of experimental evidence (and was recently found useful, for example, by Jaminet & Van Atta (1969) in understanding their observations). However, it is not clear just how it fits into the current picture.

Berger (1964*a*) confirmed the existence of this transition and of the occurrence of irregular vortex streets at the transitional Reynolds number. He also found that, in the Reynolds number range 125 to 160, there was a 'basic mode' or 'ground-state', which sometimes occurred instead of the 'high-speed mode'. This gave a vortex street of somewhat different frequency and of an extremely regular character.

The various expressions proposed for the dependence of S on R have not previously been collected together, and it seems useful to do so. They are shown in figure 1. The ordinate has been chosen as S, as this allows an expanded scale to bring out the differences; however, it should be noted that, in every case except Tritton's expression for the low speed mode, the data were represented by a linear relationship between R and F. For the high-speed mode the expressions proposed by Berger and Tritton are identical. For the low-speed mode they are slightly different. Unfortunately, the difference, although minor, is, as we shall see in a moment, most marked in the region where the details affect the interpretation.

Berger considers it possible, though not certain, that the 'basic mode' is an extension of the 'low-speed mode'. Whether the latter can be extrapolated in a way that would join smoothly onto the former depends on whose formula is used. With Berger's expression it is possible although not automatic; with Tritton's it is scarcely possible (figure 1). The data on which Tritton's equations were based has been re-examined. It would be hard to fit these data to Berger's equations. In particular, in the region of overlap, the two modes fit much better to two nearly parallel lines than to two intersecting lines. However, there is no reason to suppose that these data are superior to Berger's, and it would certainly simplify the interpretation to suppose that the 'low-speed mode' and the 'basic mode' are different régimes of the same flow behaviour.†

† *Note added in proof:* Since preparation of this paper, Dr Berger has drawn my attention to his more recent observations (Berger 1964*b*) showing these two modes joining continuously.

The current situation is further complicated by Gaster's (1969) report that he finds no discontinuity. He considers it valid to use Roshko's equation to represent the behaviour throughout the range.

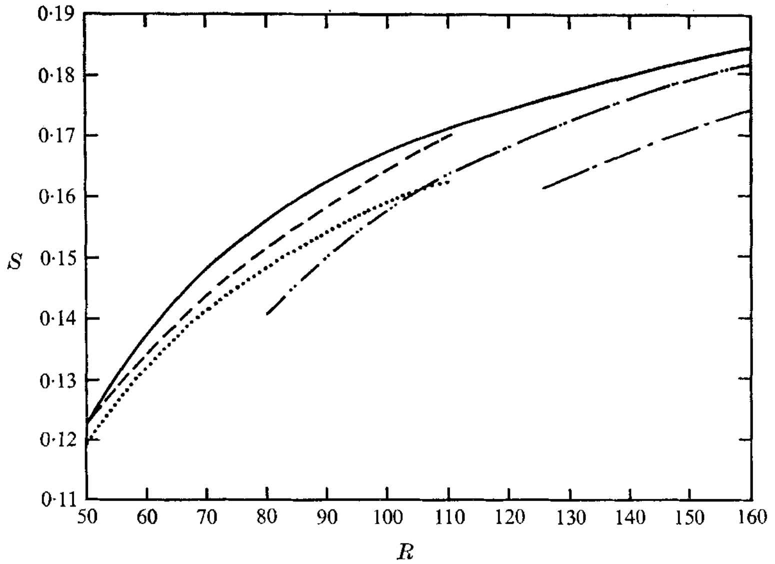

FIGURE 1. Various proposals for relationships between Reynolds number and Strouhal number. ——, original proposal by Roshko $S = 0{\cdot}212 - 4{\cdot}5/R$; – – –, Tritton's proposal for low speed mode, $S = 0{\cdot}144 - 2{\cdot}1/R + 0{\cdot}00041R$; · · · · ·, Berger's proposal for low-speed mode, $S = 0{\cdot}197 - 3{\cdot}9/R$; · · — · ·, high-speed mode (as given by both Tritton and Berger), $S = 0{\cdot}224 - 6{\cdot}7/R$; — · —, Berger's basic mode, $S = 0{\cdot}220 - 7{\cdot}4/R$.

3. Objections to Gaster's interpretation

The non-occurrence of irregular vortex streets in Gaster's experiments must evidently be taken into account in any interpretation of the data. On the other hand there are objections to his specific explanation of Tritton's observation of irregularities: he notes similarities between the irregular streets and vortex streets behind tapered cylinders and so suggests that the observation was the result of some non-uniformity in the flow.

In addition to Berger's confirmation of the observation (and it should perhaps be pointed out that, although the caption to Berger's figure 24 refers to Tritton, the figure shows Berger's own photographs of irregular vortex streets), the following points need to be made.

Regular vortex streets occur at values of the Reynolds number both below and above the transition. Since, in Tritton's original experiments, the transition occurred at similar values of the Reynolds number for different cylinder diameters, the irregular vortex streets were not associated with some particular speed of the wind tunnel.

Again in these original experiments, irregular vortex streets were observed in both a wind tunnel and a water channel. It would be an unlikely coincidence for both these to develop flow non-uniformities over just a short speed range.

The existence of the transition was indicated in the first place by the discontinuity in the Reynolds number–Strouhal number relationship. Thus, even if an alternative explanation of the irregular vortex streets were tenable, there would still be evidence for the transition.

4. A further experiment

Despite these considerations, it seemed worthwhile to do a brief experiment that was essentially a repeat of Tritton's (1959) experiments in a different wind tunnel. This was intended only to test the repeatability of the observation of the transition, and was done with as little elaboration as was consistent with that aim.

The wind tunnel was an old one of the type described by Salter (1947). A cylinder of diameter 2·35 mm was placed across the entry to its working section. The vortex street behind this was examined by conventional hot-wire techniques and its frequency measured by obtaining a Lissajous figure between the signal from the hot-wire and one from an oscillator; the latter was calibrated using a timer/counter.

The wind-tunnel speed was not measured absolutely, but was monitored using a DAVImeter (Direct Air Velocity Indicating meter) supplied by Airflow Developments Ltd. of High Wycombe. The sensitive element of this is a heated semiconductor bead. Sufficient sensitivity could not be obtained using the instrument's own meter. Hence, an electronic d.c. millivoltmeter was connected directly in parallel with the instrument's meter. This resulted in both sufficient sensitivity and sufficient repeatability (at least within a single run).

The results are shown in figure 2. A discontinuity is evident in the vicinity of $F = 18$. That it can be identified with the discontinuity reported in Tritton (1959) is confirmed by the occurrence of regular vortex streets both below and above the discontinuity but irregular ones in the transition region (figure 3).

The value of F suggests that the transition was occurring at a Reynolds number of about 110, rather above the range of values of the earlier experiments.

The detailed distribution of the points in the vicinity of the transition suggests that the R, S relationships for the two modes may intersect, but at a markedly higher Reynolds number than implied by Berger's equations. The experiment is thus indecisive on the question of whether Tritton's or Berger's equation for the low-speed mode is the better representation.

No sign of anything that could be interpreted as Berger's 'basic mode' was found; however, the wind tunnel almost certainly has a high background turbulence level so this is not surprising.

5. Discussion

Evidently there is some variability in the detailed behaviour of the flow: the flow patterns within the transition exhibit differences in detail (Tritton 1959); the Reynolds number of transition can vary somewhat; Berger's 'basic mode' may or may not occur; maybe, depending on how one interprets Gaster's observations (see below), the transition is sometimes omitted altogether. Pre-

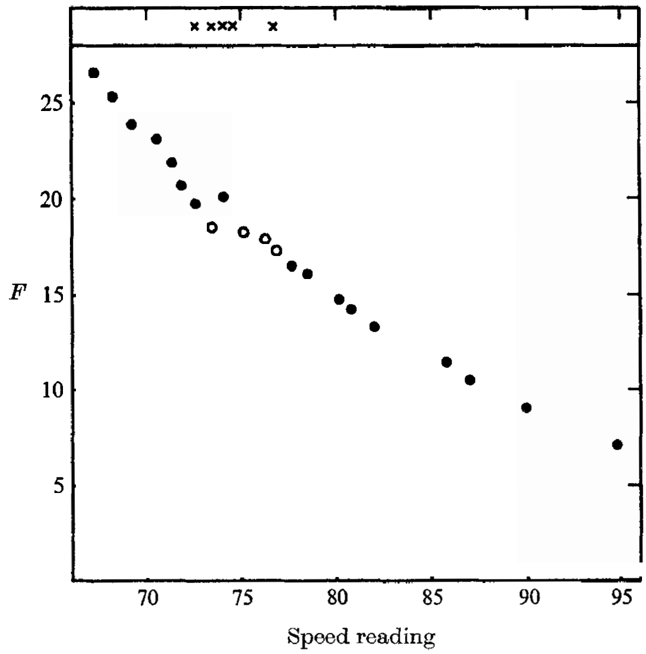

FIGURE 2. Observations of vortex street frequency (expressed as $F = nD^2/\nu$) as speed was varied. Speed reading is meter reading in arbitrary units decreasing as speed was increased (not necessarily linearly). Closed circles indicate steady Lissajous figures. Open circles indicate Lissajous figures that were not steady but which had sufficient steady periods for a frequency to be determined. The crosses at the top indicate speed settings for which the figure was so unsteady that no frequency could be determined (the left-hand one because the vortex street was becoming turbulent). All observations were made 40 diameters downstream from the cylinders.

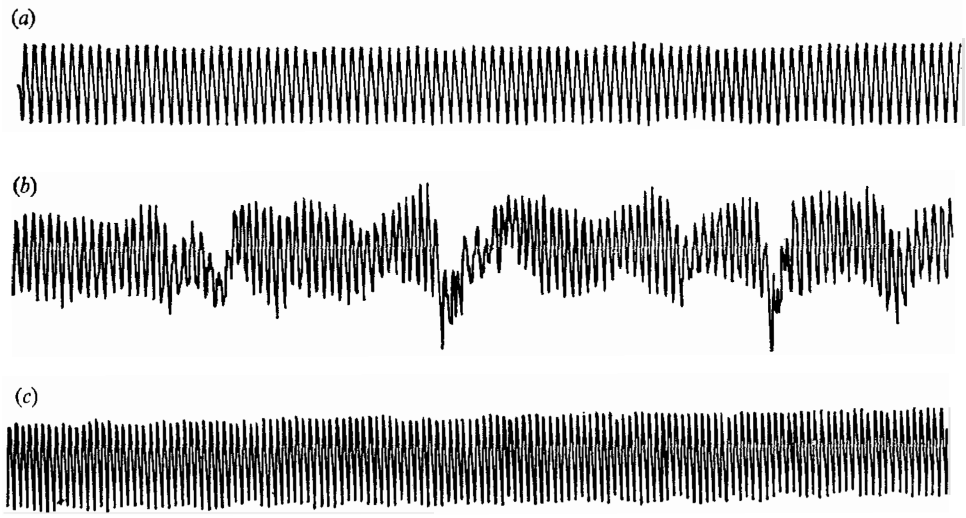

FIGURE 3. Oscillograms of velocity fluctuations, 40 diameters downstream of cylinder. Time increases to right, velocity increases upwards. (*a*) Low-speed mode, speed setting (cf. figure 2) = 77·5. (*b*) Transitional flow, speed setting = 73·9. (*c*) High-speed mode, speed setting = 70·4.

sumably the exact behaviour is governed by the extent and nature of uncontrollable small departures from the ideal situation—background turbulence, slight non-uniformity of the cylinder, slight non-circularity of the cylinder, etc. The details of many other transitional processes are similarly affected. This does not imply that the transition itself is a consequence of such departures. In my view, it can now be regarded as established that there are different modes of the vortex street between which transition can occur.

This leaves the question of why Gaster did not observe any transition. Three alternative possibilities come to mind (though in my opinion the third is unlikely).

(i) The transition from one mode to the other may have been occurring but the speed was never just right for the transitional type of flow. The Reynolds number range of transition can be sufficiently short that it is sometimes necessary to make a velocity–frequency plot before the velocity can be set to give transitional flow. A detailed study of Gaster's figure 5 suggests that the frequency measurements are not inconsistent with the transition occurring provided it occurs behind both the straight and the tapered cylinder. When the local diameter of the tapered cylinder is close to the diameter of the straight cylinder the transitions need not show on a graph that amounts to a plot of the frequency behind one against the frequency behind the other (as the speed is varied). When the diameters are significantly different, there should be signs of two small discontinuities. It is possible to fit such an interpretation to the data.

(ii) If the 'low-speed mode' and the 'basic mode' are the same thing, it may be possible for the flow to remain in this mode throughout the Reynolds number range 50 to 160.† If this is the explanation, it needs to be noted (figure 1) that the 'basic mode' has a Reynolds number–Strouhal number relationship significantly different from Roshko's equation. Thus Gaster's calibrations would be in error.

(iii) Since the Reynolds number of the transition is rather variable, it is possible that it can sometimes go above 160 (when all the dependences break down because of instability of the vortex street). This would imply a range of Reynolds number in which any of three possibilities ('low-speed mode', 'high-speed mode', or 'basic mode') can occur.

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† *Note added in proof:* Berger's more recent observations (see footnote in §2) make this much the most likely explanation.